Written Exam at the Department of Economics winter 2018-19

Economics of the Environment, Natural Resources and Climate Change

Final exam

21 December 2018

(3 hour closed book exam)

Answers only in English

This exam question consists of 5 pages in total, including this front page.

NB: If you fall ill during an examination at Peter Bangs Vej, you must contact an invigilator who will show you how to register and submit a blank exam paper. Then you leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

Be careful not to cheat at exams! You cheat at an exam, if during the exam, you:

- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

Exercise 1. Optimal climate policy in a simple growth model (indicative weight: 3/4)

Consider a simplified model of economic growth and climate change that uses the following notation:

Y = gross output before damage from climate change

g = growth rate of gross output (constant)

C = consumption of final goods

M = total use of energy

m = energy use per unit of gross output

a = share of carbon-free energy in total energy use

E = emission of CO_2

q = cost of one unit of carbon-free energy

c = cost of one unit of fossil-based energy

z = average unit cost of energy

τ = carbon tax rate

S = stock of CO_2 accumulated in the atmosphere

D = fraction of output lost due to damages from climate change

W = social welfare

ρ = rate of time preference (constant)

δ = rate of decay of carbon stock in the atmosphere (constant)

t = time

The economy is in a steady state where gross output increases at the constant rate g and where we set the initial level of output equal to 1 for convenience:

$$Y_t = e^{gt}, \qquad g > 0. \tag{1}$$

By definition, total energy use equals the energy use per unit of output multiplied by total output:

$$M_t = m_t Y_t. \tag{2}$$

Emissions of CO_2 are proportional to the total use of fossil-based energy, and we can set the proportionality factor equal to 1 by appropriate choice of units. The share of fossil-based energy in total energy use is $1 - a_t$, so total CO_2 emissions are

$$E_t = (1 - a_t)\, M_t, \qquad 0 \leq a_t \leq 1. \tag{3}$$

As the share a_t of fossil-free energy in total energy use increases, it becomes more and more costly to increase it even further. On the other hand, due to technical progress in green energy technologies, the unit cost of fossil-free energy decreases at the rate α for any given value of a_t. Thus we assume that the cost q_t of producing a unit of fossil-free energy is given by

$$q_t = q_0 \frac{a_t^{\eta-1}}{\eta} e^{-\alpha t}, \qquad \eta > 1, \tag{4}$$

where q_0, η and α are constants. The cost of producing a unit of fossil-based energy is c_t, and fossil energy is subject to the carbon tax τ_t, so the average unit cost of energy for firms and households is

$$z_t = a_t q_t + (1 - a_t)\,(c_t + \tau_t)\,. \tag{5}$$

To keep the model simple, we abstract from capital accumulation. Hence the total consumption of final goods equals gross output minus the total cost of energy production and minus the damage cost of climate change:

$$C_t = Y_t - \overbrace{[a_t q_t + (1 - a_t)\, c_t]\, M_t}^{\text{Total cost of energy production}} - D_t Y_t. \tag{6}$$

Note that the carbon tax does not reduce consumption because the tax revenue is assumed to be recycled as a lump sum transfer to consumers. The damage cost per unit of gross output is assumed to be proportional to the accumulated stock of carbon in the atmosphere (S_t) which drives global warming:

$$D_t = \gamma S_t. \tag{7}$$

The damage cost parameter γ is treated as a constant. Over time, a constant (small) fraction $\delta > 0$ of the existing stock of CO_2 in the atmosphere is absorbed by other carbon reservoirs, but at the same time new emissions add to the carbon stock S_t which therefore evolves as

$$\dot{S}_t = E_t - \delta S_t. \tag{8}$$

This completes the description of the model.

Question 1.1. In each period firms and households choose their mix of fossil-free and fossil-based energy with the purpose of minimizing their total unit cost of energy, taking q_t, c_t and τ_t as given. Use (4) and (5) to show that the cost-minimizing share of fossil-free energy is

$$a_t = \left(\frac{c_t + \tau_t}{q_0 e^{-\alpha t}}\right)^{\varepsilon}, \qquad \varepsilon \equiv \frac{1}{\eta - 1} > 0. \tag{9}$$

Explain the economic intuition behind the result in (9). (Note that you do not need to solve an optimal control problem at this stage; you only need eqs. (4) and (5) to answer the present question).

In the following you will be asked to characterize the optimal climate policy. For this purpose, we assume that the social planner wishes to maximize the following objective function which defines social welfare as the present value of future consumption:

$$W = \int_0^\infty C_t e^{-\rho t} dt, \qquad \rho > g. \tag{10}$$

Question 1.2. Use the relevant equations from the model above to show that the social welfare function (10) can be written as

$$W = \int_0^\infty \left\{1 - m_t \left[q_0 \frac{a_t^\eta}{\eta} e^{-\alpha t} + (1 - a_t)\, c_t\right] - \gamma S_t\right\} e^{-(\rho - g)t} dt. \tag{11}$$

Explain why the assumption $\rho > g$ made in (10) is important.

Question 1.3. Use the relevant equations from the model above to show that the stock of carbon in the atmosphere evolves as

$$\dot{S}_t = (1 - a_t)\, m_t e^{gt} - \delta S_t, \qquad S_0 \text{ given.} \tag{12}$$

Question 1.4. The optimal climate policy is the time path for the share of fossil-free energy a_t that will maximize social welfare (11) subject to (12). Set up the current-value Hamiltonian for this optimal control problem where a_t is the control variable and S_t is the state variable (denote the shadow price of S_t by λ_t).

Question 1.5. Use the first-order condition for a_t in the optimal control problem defined in Question 1.4 to derive an expression for the socially optimal choice of the share of fossil-free energy at any given time t, written as a function of c_t and λ_t. Give an economic interpretation of this expression for a_t (note that since a higher value of S_t implies greater damage from climate change, we have $\lambda_t < 0$).

Question 1.6. Use your results in questions 1.1 and 1.5 to derive an expression for the optimal carbon tax rate τ_t as a function of the shadow price λ_t. How does the optimal carbon tax rate depend on the evolution of gross output? Explain the intuition behind your result.

Question 1.7. Go back to the optimal control problem defined in Question 1.4 and derive the first-order condition for the optimal change in the shadow price of S_t over time ($\dot{\lambda}_t$). Show that this first-order condition implies that

$$\lambda_t = -\int_t^{\infty} \gamma e^{-(\rho-g+\delta)(u-t)} du = \frac{-\gamma}{\rho - g + \delta}. \tag{13}$$

Explain the economic intuition for the result in (13). (Hint: You may use Leibniz' Rule to prove the result in (13)).

Exercise 2. The debate on Integrated Assessment Models (indicative weight: 1/4).

(Hint: You may provide purely verbal answers to the questions in this exercise, but you are also welcome to include equations if you find it useful).

Question 2.1: Describe (briefly) the main features of a typical Integrated Assessment Model of the economy and the climate system such as the DICE model.

Question 2.2: Discuss some strengths and weaknesses of the DICE model.